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NEW INVESTIGATION ON THE AIR JET GENERATOR FOR ACOUSTIC WAVES

BY

JUL. HARTMANN AND BIRGIT TROLLE

KØBENHAVN
HOVEDKOMMISSIONÆR: ANDR. FRED. HØST & SØN, KGL. HOF-BOGHANDEL
BIANCO LUNOS BOGTRYKKERI
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Introduction.

In an earlier paper[1] the principle of the air jet generator for the production of acoustic waves, especially high-frequency waves, was explained at length. In the same paper a series of the fundamental experiments was described. While these experiments illustrate the conditions for the creation of the waves, they do not throw much light on the working-manner of the generator. Hence it seemed of interest to make this mode of working the object of a special investigation. The present paper contains the results of a research-work carried out with the aim just indicated. With a view to orientation a short description of the generator is given in the first pages of the paper and some few observations, which were not given in the original communication, are added to this description.

The air jet.

The fundamental member of the generator is an air jet of a velocity equal to or greater than the velocity of sound. A jet of this kind is created when air is emitted to the free atmosphere from a container in which the absolute pressure exceeds 1.9 atm., the over-pressure thus 0.9 atm. The jet shows a peculiar structure of which a picture may be obtained by means of the so-called "Schlierenmethode".

[1] Det kgl. danske Vidensk. Selsk. math.-fys. Medd. I, 13, 1919; Physical Review. Vol. XX, 719, 1922.

Fig. 1 *a* shows a photograph procured in this way. As will be seen, the jet is divided into sections of nearly the same

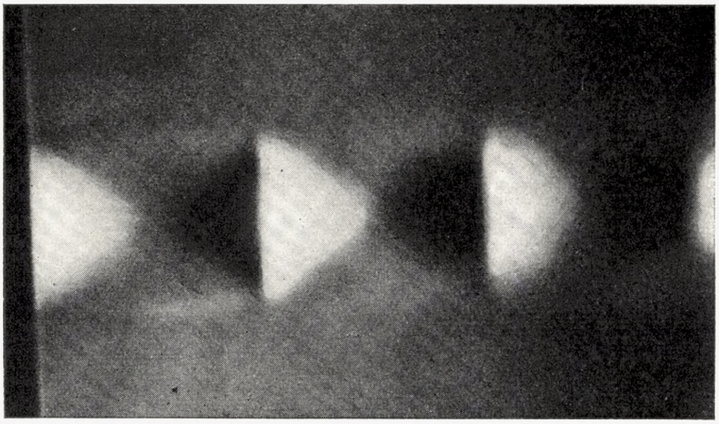

Fig. 1 *a*.

Fig. 1 *b*.

length. Corresponding to this division the static pressure varies, as shown for instance by Stodola[1], in a regular periodic way out along the axis of the jet, exhibiting minima in the middle-points of the sections and maxima in the planes separating the sections.

[1] A. Stodola. Die Dampfturbinen 1910 p. 85.

Prandtl has derived the formula

1° $$\Delta = 1.2 \cdot d \sqrt{p - 0.9}$$

for the length Δ of the sections of a cylindrical jet emitted into the free atmosphere. In the formula d denotes the diameter of the jet-hole and p the over-pressure in the container measured in atmospheres. The formula agrees fairly well with observations, the constant 1.2 being perhaps somewhat too high. Moreover the first section is found to be of somewhat greater length than the following sections, namely about $^8/_7$ of these.

The Pitot curve.

If the sound of a simple Pitot apparatus, fig. 2, is moved along the axis of the air jet, the aperture being directed against the stream, the reading of the apparatus will vary in a regular way. Close to the jet-hole the pitot apparatus indicates the pressure which is found in the container from which the air is emitted. When the sound is gradually removed from the nozzle the successive pressure observations will generally prove to arrange themselves according to a curve of damped oscillations as shown in fig. 3. In this graph the abscissae indicate the distance of the mouth of the sound from the nozzle, the reading 100 corresponding to 0.275 mm. The diameter of the jet-hole was 0.535 mm. and the over-pressure in the container

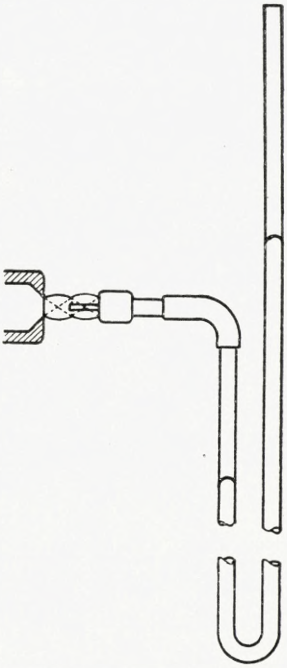

Fig. 2.

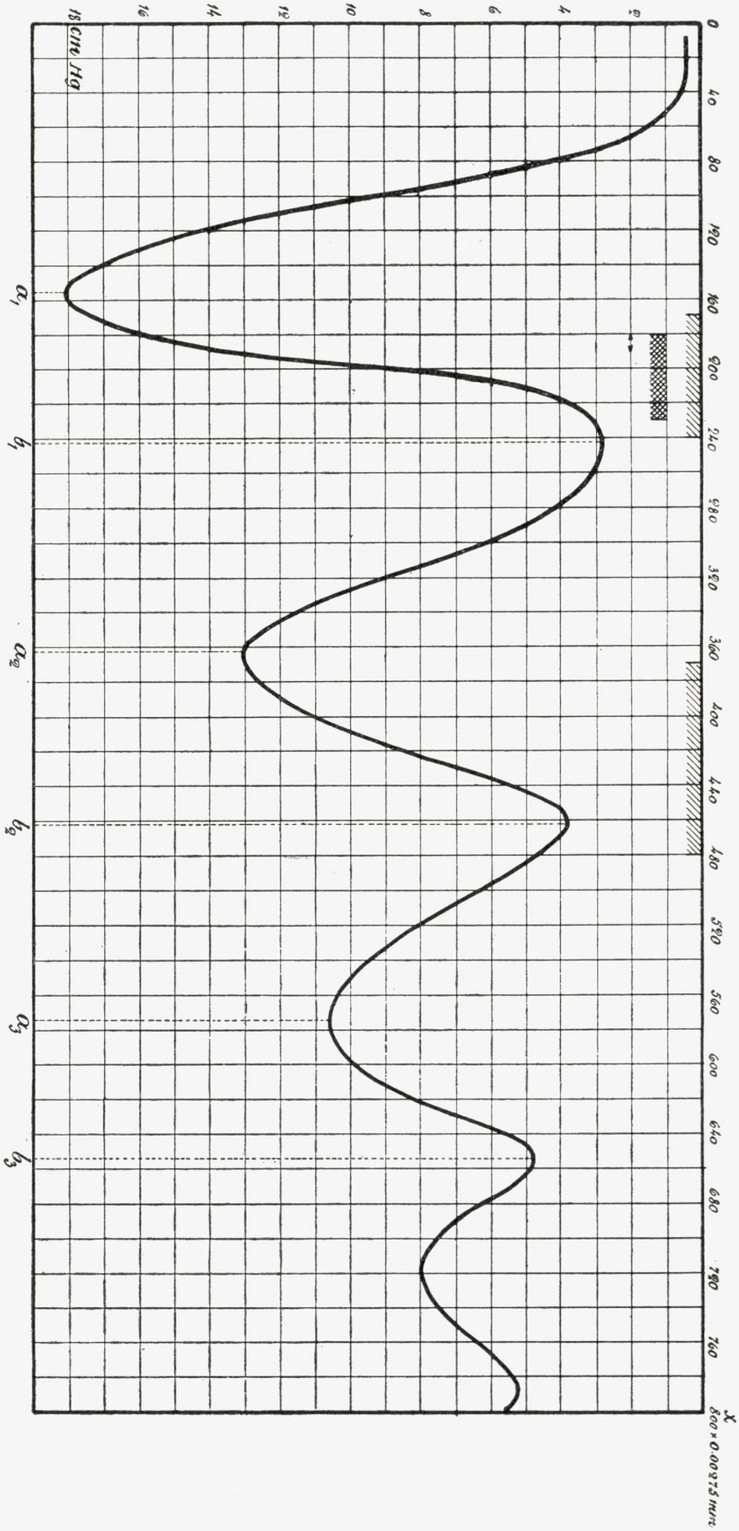
0
40
80
120
160
200
240
280
320
360
400
440
480
520
560
600
640
680
720
760
x
800 × 0.00275 mm
2
4
6
8
10
12
14
16
18 cm Hg
a1
b1
a2
b2
a3
b3

Fig. 3.

was 141 cm. Hg. While engaged in the survey of the jet by means of the pitot apparatus, one of the authors found that, unless particular precautions were taken, it was practically impossible to read the pitot manometer inside those intervals where the pitot curve is rising, i. e. the intervals marked a_1 b_1 a_2 b_2 etc. in fig. 3. In these intervals the manometer behaved in the following manner. First it took in air to a certain pressure, then discharged the air, then again took in air to the same pressure as before, discharged etc. This observation gave rise to the discovery that the air jet with a velocity greater than that of sound constitutes a means for the production of acoustic oscillations and waves.

The air jet generator for acoustic waves.

Instead of the sound of the pitot apparatus the mouth of a container, as shown in fig. 4, was introduced into the jet. When the mouth was adjusted in one of the intervals indicated above — the intervals of instability — the container took in and discharged air periodically just as the pito-manometer. The period of the "pulsation" depends on the capacity of the container, the "pulsator", on the diameter of the inlet of the same, and on the position of the mouth of the pulsator in the interval of instability. The larger the capacity and the narrower the inlet the longer the period, which may easily be varied from many seconds or even minutes to $^1/_{1000}$ of a second or less. The pulsations may, with low frequency, be made visible by placing a

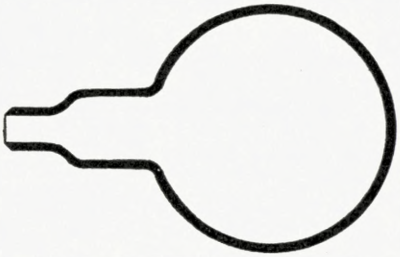

Fig. 4.

strip of paper on one side of the mouth of the pulsator, the strip being flung violently outwards at every discharge.

A number of instantaneous photographs of the pulsations were taken. The jet was a hydrogen jet. In fig. 5 *a—d* four of the

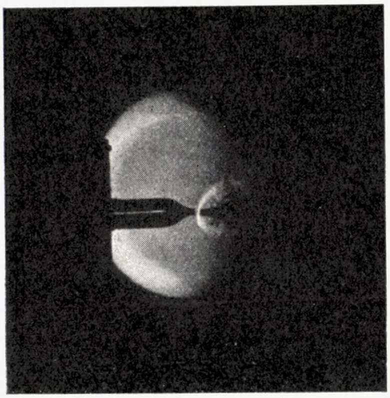

Fig. 5 *a*.

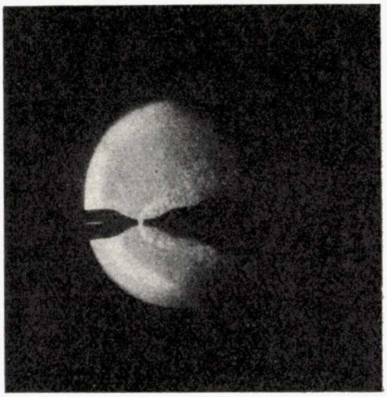

Fig. 5 *b*.

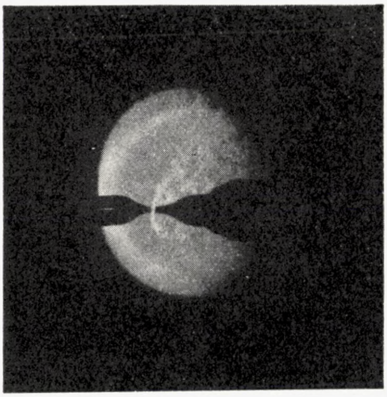

Fig. 5 *c*.

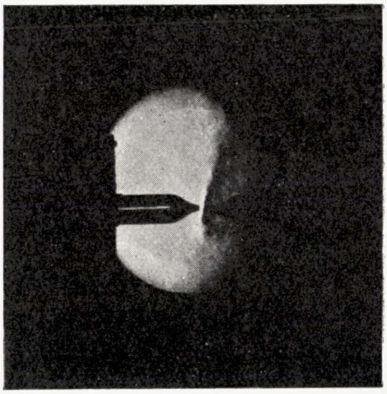

Fig. 5 *d*.

photographs are reproduced. Fig. 5 *a—c* presumably represent three successive instants during the filling of the pulsator. The cloud which appears in the pictures indicates the vortices, formed by the surplus of air which does not enter the pulsator. Fig. 5 *d*, undoubtedly represents the conditions during the discharge. The latter is brought about by an air jet directed against the main jet emitted from the nozzle. The collision between the two jets gives rise to a violent compression as seen clearly in the picture. Of the great number of photographs taken most were of the types shown in fig. 5 *a—c*. From this experience it was concluded that the phase of discharge was, in the case considered, essentially shorter than that of the filling.

A close examination of the plate of which fig. 5 *a* is a reproduction showed that the surroundings of the nozzle of the pulsator were filled with sound waves with a wave-length corresponding to ab. 350000 vibrations per sec. Undoubtedly the waves originate from the narrow mouth of the pulsator acting as an oscillator (see below). The aperture of the mouth was ab. 0.6 mm.

If the pulsator is replaced by a cylindrical oscillator consisting, as indicated in fig. 6, of a cylindrical bore in the pointed end of a short cylindrical piece of brass, one gets acoustic vibrations of a frequency which, provided the depths of the bore exceeds a certain limit, nearly equals the natural frequency of the oscillator. But, again, the vibrations do not appear unless the mouth of the oscillator is adjusted in one of the intervals of instability. The frequency is, however, not quite independent of the position in the interval. The variations are especially pronounced with oscillators of smaller depth, as will appear from the experiments which are the main object of this paper.

Fig. 6.

The position of the intervals of instability relatively to the pitot curve.

As stated above, the intervals of instability coincide with those parts of the pitot curve which correspond to a rising pressure indicated by the pitot manometer. In fig. 3 the intervals of instability are marked by hatching beneath the axis of abscissae. The intervals were determined by means of small pulsators. In the determination of the first interval a pulsator of 1.5 cm^3 capacity was first applied. It gave the interval indicated by the uppermost hatching. With a pulsator of a few mm^3 capacity the interval below was found. If the mouth of the latter pulsator was adjusted in the interval indicated by the twin-arrow, the mouth acted as an oscillator emitting waves with a fre-

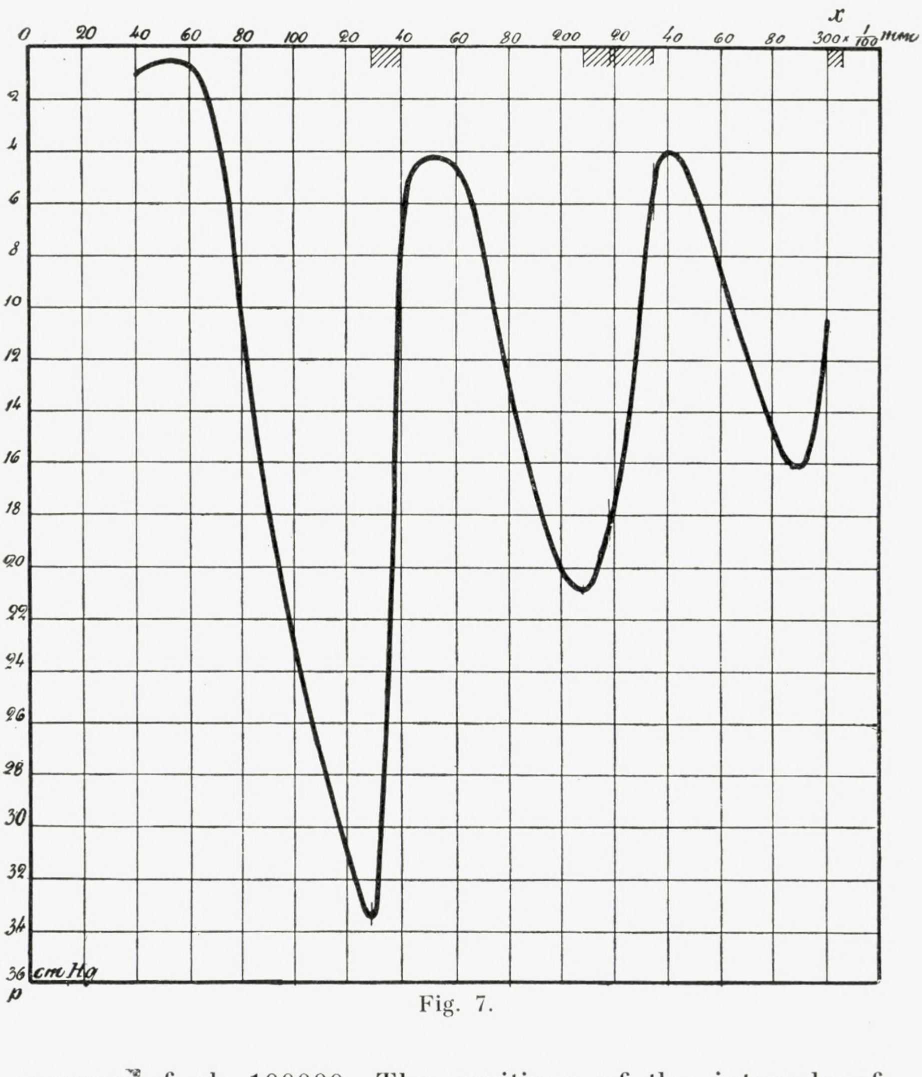

Fig. 7.

quency of ab. 100000. The positions of the intervals of instability were not determined simultaneously with the pitot-curve. The positions relatively to the latter are therefore somewhat uncertain, owing to comparatively small alterations in the pressure of the air in the container giving rise to relatively large displacements of the curve. Fig. 7 represents a later and more exact determination in which the intervals were observed at the same time as the curve. The intervals, obviously, coincide very precisely with the rising branches of the curve.

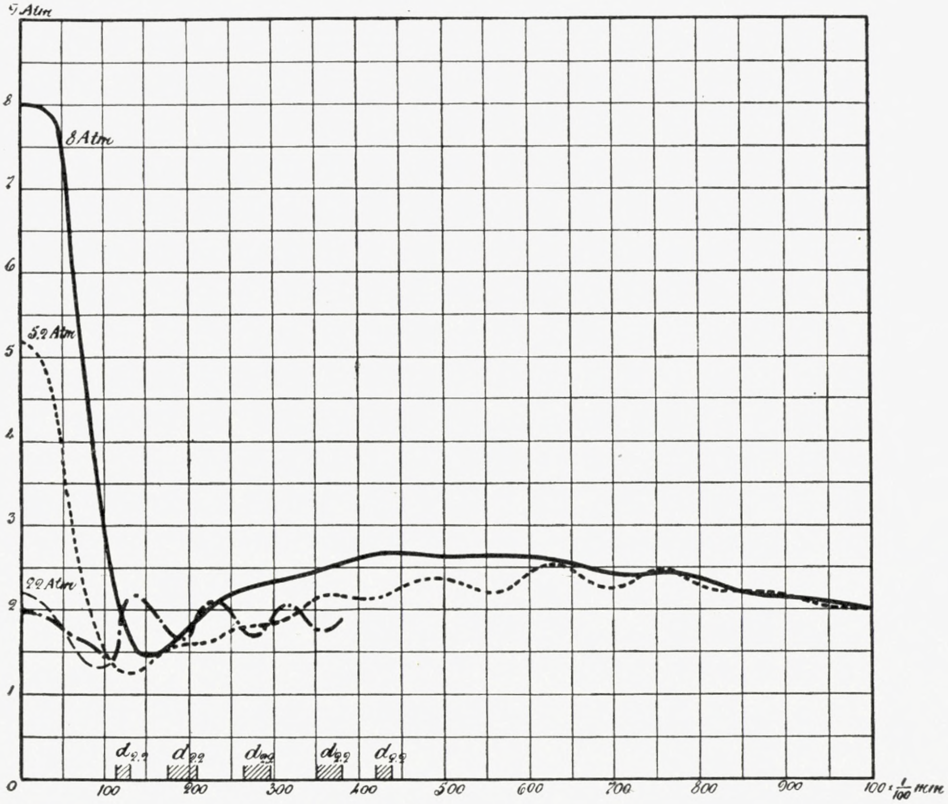

Fig. 8.

If the pressure in the container is increased the wave-length of the pitot curve at first increases corresponding to the lengthening of the sections of the jet predicted by Prandtl's expression above. If, however, the pressure is raised above a certain limit, the character of the pitot curve changes, the periodic, damped variations disappearing. The changes of the curve can be followed in fig. 8 representing three pitot curves corresponding to pressures of ab. 2.2, 5.2 and 8 atm. and to a jet-hole of 0.7 mm. Simultaneously with the disappearance of the periodic variations, a change of the structure of the jet seems to occur, the jet assuming the aspect shown in fig. 1 *b*. The transition is furthermore characterized by the intervals of instability spreading out and ultimately overlapping. After this has happened, one may, within continuous parts of considerable length, obtain acoustic vibrations by means

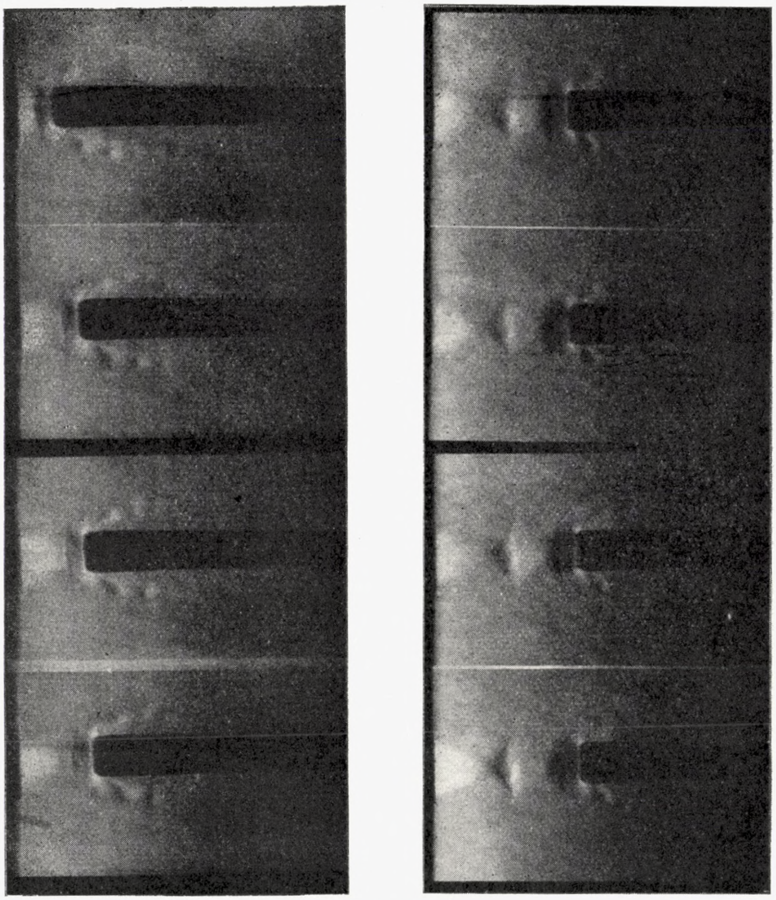

Fig. 9.

of a pulsator or oscillator. Investigations on this matter have been described in the first communications. In the present paper we mainly confine ourselves to the consideration of jets of which the pitot curve is of the periodic type.

The position of the pitot curve relatively to the jet-picture.

Above the intervals of instability have been mapped out relatively to the pitot curve. There can be but little doubt that the latter coincides with the jet picture in such a way that the minimae of the curve are to be found at the points where, according to Stodola, the photograph of the jet indicates the presence of minimae in the static pressure, i. e. in the middle-points of the sections, and maximae in the places where the static pressure assumes maximum values i. e. in the borders of the jet sections. However, it seemed best to obtain experimental evidence with regard to this point, thus to fix the position of the pitot curve exactly in relation to the jet picture. With a view to doing

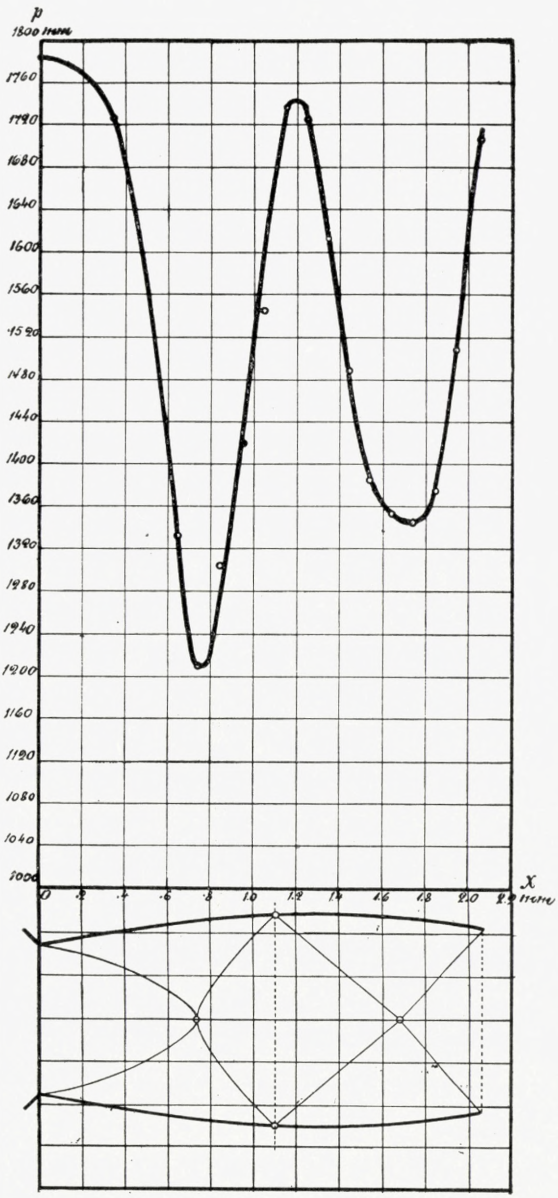

Fig. 10.

this, curves were observed and simultaneously photographs of the jet were taken for a number of positions of the pitot sound. In fig. 9 two of the photographic plates, each with four pictures, have been reproduced. On the basis of the photographs and the pitot observations the pitot curve could be drawn, together with a picture of the jet in the right position relatively to the latter. In fig. 10 a drawing of this kind is shown. It corresponds to a jet produced by means of an obtuse conical bore of 0.766 mm. diameter. The over-pressure was 2.48 atm. Evidently the minimum of the pitot curve is almost exactly above the cross-point of the jet picture i. e. the spot in which the static pressure is minimum. It should be noted that the abscissa of the pitot curve indicates the position of the mouth of the pitot sound.

The relation between the wave-length and the position of the oscillator in the interval of instability.

We now proceed to describe a series of new experiments on the relation between the wave-length and the position of the oscillator within the interval of instability. Only oscillators of small depth were used. The wave-length was measured by the Kundt-tube method. In fig. 11—13, in which d indicates the aperture of the oscillator and l the depth of the same, the results of three of the preliminary experiments are reproduced. The abscissa indicates the distance x from the nozzle to the mouth of the oscillator. The ordinate denotes a quarter of the observed wave-length. On the axis of abscissae the boundaries S_1 S_2 S_3 of the jet sections are marked. Considering first fig. 11, we shall observe that the wave-length, within each interval of instability, increases linearly with the distance of the oscillator from the entrance of the jet section — the curves A_1 and A_2. At a certain distance the wave-length suddenly drops after which it again increases almost linearly, but not so steeply as before, the curves B_1 B_2. The two regions somewhat overlap i. e. within a certain region it is a matter of chance which of the two wave-lengths occurs. The waves corresponding to the curves A_1 and A_2 are by far the most regular and intense. In the following they alone are considered. The A-curves have, at any rate in fig. 11, very nearly an inclination of 45° against the axis of abscissae. This means that the quarter-wave-length increases with an amount just equal

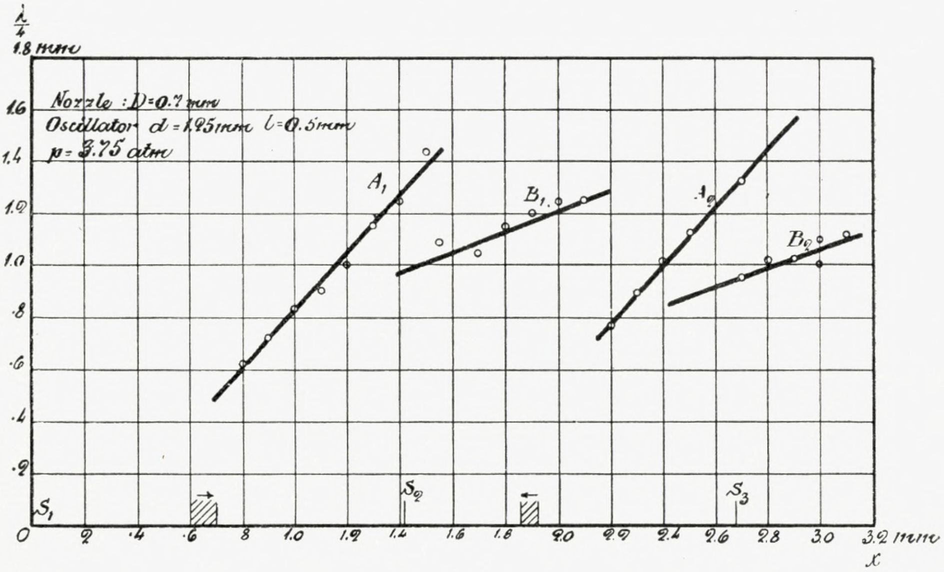

Fig. 11.

to the displacement of the oscillator when the latter is moved along the axis of the jet. Or, again, that the loop of the oscillations remains approximately in a definite spot or section of the jet. The constant position may be determined by marking off the observed quarter-wave-lengths from the bottom of the oscillator in the direction of the nozzle. The ends of these lines do not actually coincide at a definite point but are found within the comparatively narrow intervals indicated by hatching. The arrows above the intervals indicate the direction in which the loop moves when the oscillator is displaced in the direction of the axis away from the nozzle. As a rule the loop somewhat follows the displacement of the oscillator. It is thus anomalous when the opposite is the case in the second interval of fig. 11. It will be noted that the almost fixed positions of the loops in fig. 11 are to be found near the middle-points of the jet sections or rather a little in front of the same, thus nearer to the foremost boundaries of the jet section[1].

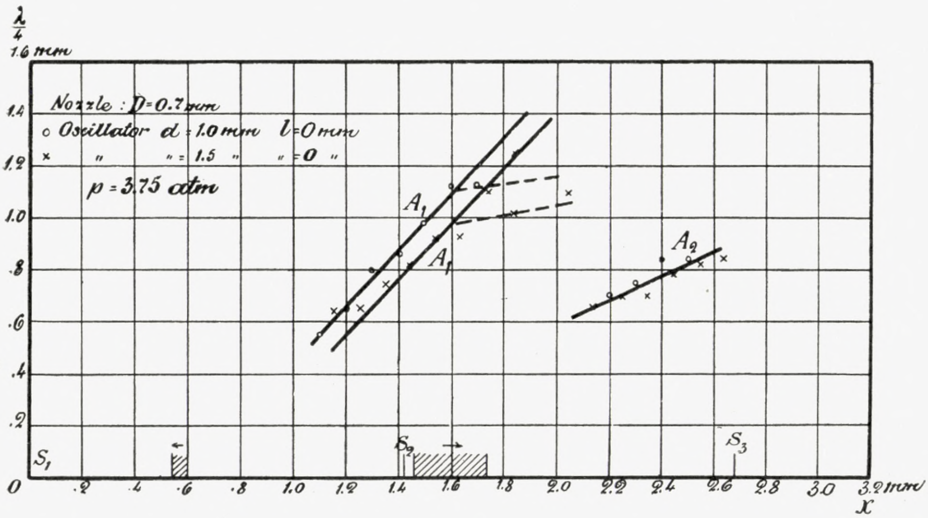

Fig. 12.

The regularity of fig. 12 and fig. 13 is not so pronounced as that of fig. 11. The experiments originate from oscillators of no depth. That means that the jet was directed against the plane end of a small brass cylinder. The experiments show that even with such an "oscillator" waves

[1] The positions of the boundaries S_2 S_3 etc. were determined by means of Prandtls Theory, regard being taken to the correction respecting the first jet section. Unfortunately, however, the original observations of the over-pressure p, could not be used in the calculation of Δ because it proved that the pressure measurement was erroneous. From select experiments, amongst which fig. 11, Δ was therefore derived as the difference of abscissae between the A_1 and A_2 curves, and from the Δ-values the two pressures 3.75 and 2.74 Atm., which were used throughout the whole series of experiments, were calculated.

are created. But the waves are of comparatively small intensity and do not exhibit any high regularity as to the variation with the position of the oscillator within the interval of instability. The greatest regularity appears within the first jet section in which the loop of the oscillations have a fairly definite position, the latter however changing somewhat with the diameter of the brass-cylinder.

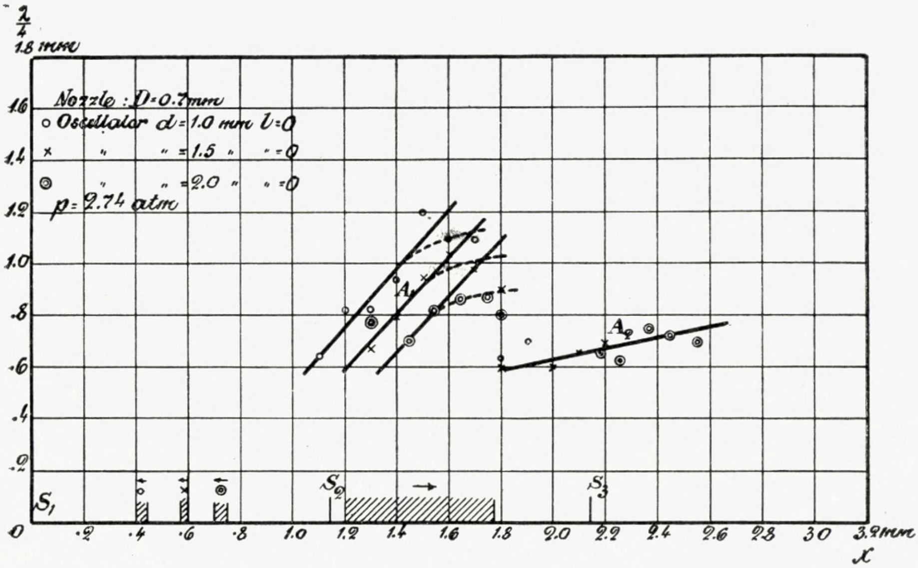

Fig. 13.

Photographs of the jet during the vibrations of the oscillator.

In a fresh series of experiments the variation of the wave-length, measured by means of a Kundt-tube, with the position of the oscillator was again studied, but, in addition, photographs of the jet during the vibrations of the oscillator were taken. In this way a more exact determination of the position of the oscillator relatively to the jet was obtained, and furthermore the photographs gave a more direct insight into the working-manner of the oscillator.

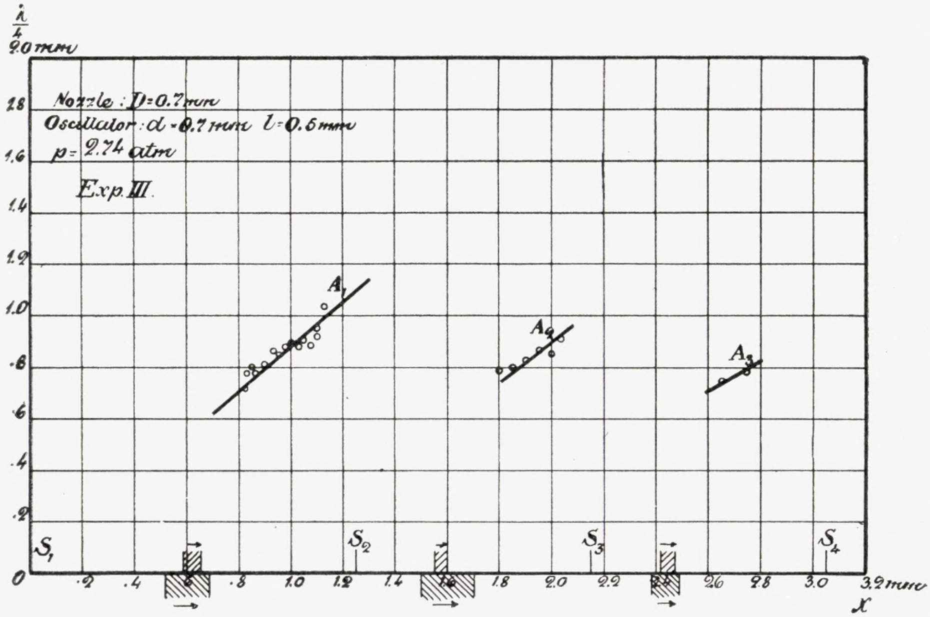
Nozzle : D=0.7 mm
Oscillator : d = 0.7 mm l = 0.5 mm
p = 2.74 atm
Exp. III.
A1
A2
A3
S1
S2
S3
S4
x

Fig. 14.

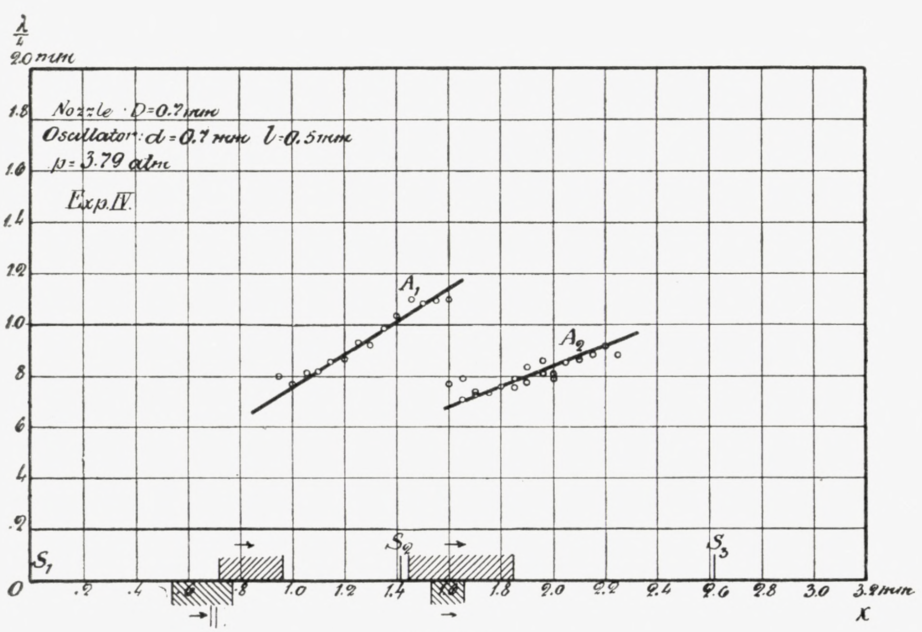
Nozzle · D=0.7 mm
Oscillator : d = 0.7 mm l = 0.5 mm
p = 3.79 atm
Exp. IV.
A1
A2
S1
S2
S3
x

Fig. 15.

In fig. 14—17 four of the Kundt-tube experiments are represented graphically in the same way as above. The A-curves and the corresponding loop intervals are shown, the latter being indicated by hatching above the axis of abscissae. Their positions are mainly the same as in the earlier experiments.

Experiment III fig. 14 exhibits the highest regularity

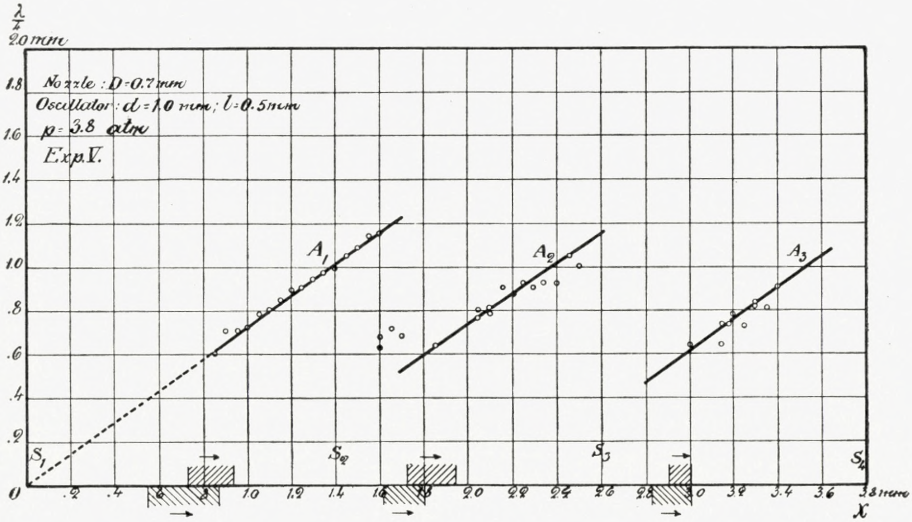

Fig. 16.

obtained. With the somewhat higher pressure in experiment IV fig. 15, the loop interval of the second jet section is rather wide, possibly due to the observations being misinterpreted — the A_2-curve being perhaps actually a B-curve, comp. fig. 11. However, the explanation may also be that the spreading out of the interval of instability, characteristic of higher over-pressures, has begun to make itself noticeable. In experiment V, fig. 16, there may again be some doubt as to the second interval. In the tracing of the curve the first observations have not been taken into account. In experiment VI fig. 17 the A-curves seem decidedly less steep. In accordance herewith the loop intervals appear rather wide.

It seemed justifiable to assume that the photographs of the jet in connection with the oscillator would prove fit for a verification of the results of the Kund-tube experi-

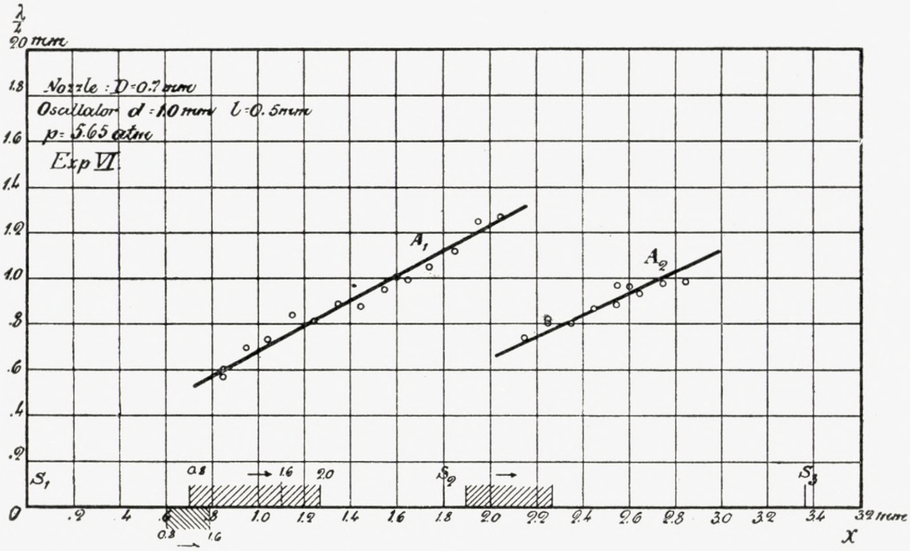

Fig. 17.

ments as to the nearly fixed position of the loop of oscillations. In the photographing, an electric spark was used as the source of light, each picture being, however, taken, illuminated not by one single spark but by a comparatively large number, namely 20—50 consecutive sparks.

Fig. 18 shows a series of photographs obtained in connection with the experiment represented in fig. 14. On considering the pictures in fig. 18 a one may observe a rather sharp line in front of the oscillator. It indicates that an abrupt change in pressure, a discontinuity of pressure, is

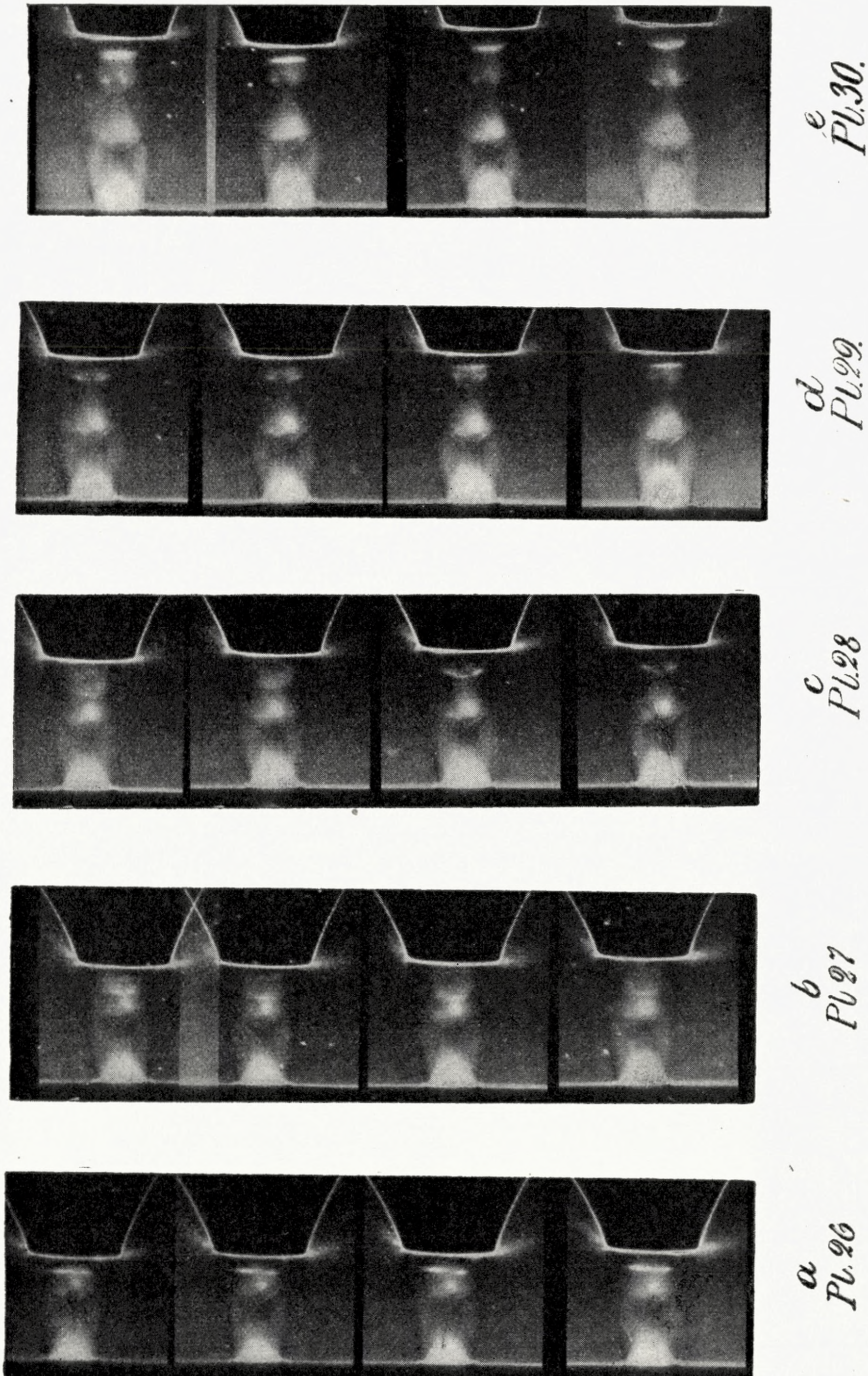

Fig. 18.

formed just before the mouth of the oscillator. If the oscillator is now moved away from the nozzle into an interval of instability the said line becomes diffuse, spreading out to a band of great or small width. The pictures in fig. 18 *b* clearly show the band. Undoubtedly, what is seen is the vibrations of the air inside and in front of the oscillator, which oscillations have thus been made visible. With regard to the interpretation various possibilities occur. The simplest explanation is perhaps that the oscillations are carried out by a definite mass of air contained mainly in the oscillator and limited by the said discontinuity. If this be the case the amplitude of the vibrations may be derived directly from the photographs in fig. 18.[1]

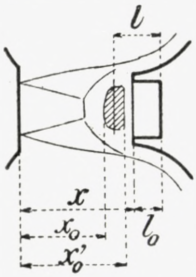

Fig. 19.

With the indications of fig. 19 we get for the amplitude of the front of the oscillating air-column $\frac{x_0' - x_0}{2}$. And, if the oscillations are regarded as stationary waves reflected from the bottom of the oscillator the quantity $l = x - \frac{x_0 + x_0'}{2} + l_0$ should be a quarter of a wave-length measured at the temperature of the oscillator. We may compare l with the wave-length of the emitted waves as found by the Kundt-tube, and we may try whether the position of the middle-point of the band of vibrations coincides with that of the loop. The middle-point of the said

[1] The oscillations need of course not be harmonic and probably are not. A picture of them might be obtained by projecting the band of oscillations on a slit parallel to the axis of the jet, a photographic film being moved with great velocity behind the slit. Undoubtedly great technical difficulties would be met with in the realisation of this plan when applied to vibrations of the frequencies considered i. e. ab. 100000. But with similar oscillators of larger dimensions the curve of oscillations might probably be obtained in the way indicated.

band has, as appears from fig. 19, a distance from the nozzle of $\frac{x_0 + x_0'}{2}$. The comparisons indicated have been made in tables I—IV below for the experiments pictured in fig. 14—17. In the figures the intervals of the middle-points of the bands of vibrations have been indicated by hatching below the axis of abscissae. In experiment III the band intervals practically coincide with the loop intervals. In the experiments with higher pressures the intervals determined from the photographs seem to be somewhat

Table I.

Experiment III. Fig. 14. Jet-hole 0.7 mm. Oscillator: $d = 0.7$ mm. $l_0 = 0.5$ mm. Over-pressure 2.74 atm. Plates No. 18—30.

No.	λ/4 Kundt-tube	x_0	x_0'	$\frac{x_0 + x_0'}{2}$	x	l	$\frac{x_0' - x_0}{2}$
	mm	mm	mm	mm	mm	mm	mm
18	0.72	0.46	0.57	0.51	0.83	0.82	0.05
19	0.78	0.44	0.60	0.52	0.85	0.83	0.08
—	0.80	0.44	0.64	0.54	0.87	0.83	0.10
20	0.78	0.42	0.62	0.52	0.88	0.86	0.10
—	0.82	0.45	0.67	0.56	0.90	0.84	0.11
—	0.86	0.43	0.70	0.56	0.92	0.86	0.14
—	0.84	0.43	0.75	0.59	0.95	0.86	0.16
21	0.88	0.44	0.76	0.60	0.98	0.88	0.16
—	0.89	0.44	0.83	0.63	1.00	0.87	0.19
—	0.87	0.46	0.88	0.67	1.03	0.86	0.21
—	0.90	0.48	0.89	0.68	1.05	0.87	0.21
22	0.90	0.50	0.82	0.66	1.08	0.92	0.16
—	0.95	0.53	0.81	0.67	1.10	0.93	0.14
—	1.04	0.55	0.78	0.67	1.13	0.96	0.12
27	0.79	1.37	1.62	1.50	1.80	0.80	0.12
—	0.80	1.33	1.65	1.49	1.85	0.86	0.16
—	0.83	1.36	1.76	1.56	1.90	0.84	0.20
—	0.87	1.38	1.87	1.62	1.95	0.83	0.20
28	0.86	1.43	1.90	1.67	2.00	0.83	0.28
—	0.91	1.51	1.89	1.70	2.05	0.85	0.19
30	0.75	2.31	2.46	2.38	2.65	0.77	0.07
—	0.78	2.41	2.52	2.47	2.75	0.78	0.06

Table II.

Experiment IV. Fig. 15. Jet-hole 0.7 mm. Oscillator: $d = 0.7$ mm. $l_0 = 0.5$ mm. Over-pressure 3.79 atm. Plates No. 31—41.

No.	$\lambda/4$ Kundt-tube	x	x'	$\frac{x_0 + x_0'}{2}$	x	l	$\frac{x_0' - x_0}{2}$
	mm	mm	mm	mm	mm	mm	mm
33	0.86	0.53	0.83	0.68	1.15	0.97	0.15
—	0.87	0.55	0.88	0.72	1.20	0.98	0.17
—	0.94	0.52	0.86	0.69	1.25	1.06	0.17
34	0.92	0.58	0.90	0.74	1.30	1.06	0.16
—	0.98	0.57	0.89	0.73	1.35	1.12	0.16
—	1.03	0.64	0.97	0.80	1.40	1.10	0.16
—	1.10	0.67	0.99	0.83	1.45	1.12	0.16
35	1.09	0.71	1.21	0.96	1.50	1.04	0.25
—	1.10	0.77	1.21	0.99	1.55	1.06	0.22
—	1.10	0.77	1.21	0.99	1.60	1.11	0.22
40	0.81	1.47	1.68	1.57	1.95	0.92	0.11
—	0.79	1.48	1.71	1.60	2.00	0.90	0.12
—	0.85	1.52	1.78	1.65	2.05	0.90	0.13
41	0.86	1.49	1.78	1.64	2.10	0.96	0.14
—	0.87	1.53	1.86	1.69	2.15	0.96	0.17
—	0.90	1.58	1.86	1.72	2.20	0.98	0.14
—	0.87	1.60	1.94	1.77	2.25	0.98	0.17

nearer to the nozzle than the Kundt-tube intervals. In accordance herewith one finds that l is on the average greater than $\lambda/4$ measured by means of the Kundt-tube. In experiment III l and $\lambda/4$ are nearly equal. As will be seen, the amplitude assumes different values in the different jet-sections and moreover varies within each section. On an average the greatest values appear within the first interval of instability in which amplitudes of up to $^1/_4$ mm. are observed.

In the interpretation of the photographs of fig. 18 given above the layer of discontinuity of pressure in front of the oscillator has in a way been considered as a piston, impenetrable to air, and in its motion following the front of the oscillating mass of air.

Table III.

Experiment V. Fig. 16. Jet-hole 0.7 mm. Oscillator: $d = 1.0$ mm., $l_0 = 0.5$ mm. Over-pressure 3.80 atm. Plates No. 42—57.

No.	$\lambda/4$ Kundt-tube	x_0	x_0'	$\frac{x_0 - x_0'}{2}$	x	l	$\frac{x_0' - x_0}{2}$
	mm	mm	mm	mm	mm	mm	mm
42	0.60	0.45	0.66	0.55	0.85	0.80	0.11
—	0.71	0.46	0.74	0.60	0.90	0.80	0.14
43	0.71	0.46	0.74	0.60	0.95	0.85	0.14
—	0.73	0.49	0.80	0.64	1.00	0.86	0.15
—	0.78	0.50	0.85	0.68	1.05	0.87	0.18
—	0.81	0.51	0.87	0.69	1.10	0.91	0.18
44	0.84	0.49	0.81	0.65	1.15	1.00	0.16
—	0.89	0.54	0.87	0.71	1.20	1.01	0.16
—	0.91	0.56	0.87	0.72	1.25	1.03	0.16
—	0.94	0.61	0.87	0.74	1.30	1.06	0.13
45	0.97	0.60	0.91	0.75	1.35	1.10	0.15
—	0.99	0.59	0.93	0.76	1.35	1.09	0.17
—	1.04	0.64	0.96	0.80	1.40	1.10	0.16
—	1.05	0.71	1.00	0.86	1.45	1.09	0.15
49	0.90	1.52	1.70	1.61	2.15	1.04	0.09
—	0.87	1.56	1.72	1.64	2.20	1.06	0.08
—	0.93	1.61	1.75	1.68	2.25	1.07	0.07
50	0.90	1.61	1.83	1.72	2.30	1.08	0.11
—	0.93	1.62	1.84	1.73	2.35	1.12	0.11
—	0.92	1.65	1.86	1.75	2.40	1.15	0.11
—	1.05	1.69	1.91	1.80	2.45	1.15	0.11
54	0.70	2.76	2.90	2.83	3.14	0.81	0.07
—	0.75	2.79	2.90	2.85	3.16	0.81	0.06
—	0.78	2.80	2.96	2.88	3.20	0.82	0.08
55	0.73	2.82	2.98	2.90	3.25	0.85	0.08
—	0.82	2.85	3.02	2.93	3.30	0.87	0.08
—	0.81	2.87	3.03	2.95	3.35	0.90	0.08
—	0.91	2.93	3.07	3.00	3.40	0.90	0.07

The correctness of this view is of course requisite for the reliability of the determination of the amplitude indicated above. The assumption seems sound because the front is the spot where the jet is stopped or at any rate checked. But of course small displacements of the layer relatively to the front of the oscillating air may

Table IV.

Experiment VI. Fig. 17. Jet-hole 0.7 mm. Oscillator: d = 1.0 mm., l_0 = 0.5 mm. Over-pressure 5.65 atm. Plates No. 58—63.

No.	λ/4 Kundt-tube	x_0	x_0'	$\frac{x_0 + x_0'}{2}$	x	l	$\frac{x_0' - x_0}{2}$
	mm	mm	mm	mm	mm	mm	mm
58	0.58	0.55	0.65	0.60	0.85	0.75	0.15
—	0.70	0.53	0.76	0.64	0.95	0.81	0.12
—	0.73	0.53	0.74	0.64	1.05	0.91	0.10
59	0.80	0.54	0.76	0.65	1.25	1.10	0.11
—	0.88	0.54	0.80	0.67	1.35	1.18	0.13
—	0.87	0.56	0.84	0.70	1.45	1.25	0.14
—	0.94	0.64	0.93	0.78	1.55	1.27	0.15

occur and, if they do occur, will give rise to corresponding errors in the determination of the amplitude and the middle-point of the oscillations.

Some reservation must in addition be taken with respect to the impenetrability of the layer of discontinuity. Undoubtedly most of the air of the jet passes the said layer but it does not accumulate behind the layer but escapes sideward. The flow of air through the layer thus only means a stationary stream which need not essentially influence the oscillations. Fig. 9 seems to throw some light on the question as to the flow through the compression-layer. In front of the Pitot-sound a pronounced layer is seen. To each side of the sound an air-jet or stream appears, exhibiting a characteristic periodic aspect, presumably indicating the same division into sections as occurs in the main jet. These secondary jets seem to be emitted from the edge of the layer of discontinuity through which edge undoubtedly the bulk of the air of the jet passes while no indication of a flow through the central part of the layer appears.

Calculations of the intensity of the oscillations etc.

On the basis of the amplitude-measurements, we may try to form an idea of the intensity of the oscillations, of the pressure variations and of the velocities occurring in the oscillator. In doing so we must however remember that the value of the amplitude derived from the oscillations of the layer of discontinuity may, for reasons stated above, in some degree be erroneous. Furthermore it should be noted that the formulae, to be used for the calculations, have been derived only for harmonic oscillations of small amplitudes, while the amplitudes of the vibrations considered are comparatively large and the vibrations themselves undoubtedly not harmonic. The estimates must therefore only be considered as roughly approximate.

With a harmonic stationary sound-wave of sufficiently small amplitude the intensity, i. e. the energy pr. cm^3 of the vibrating mass of air, is expressed by

1° . $$I = \frac{1}{4}\varrho v_0^2 = \frac{1}{4}\varrho (2\pi N a_0)^2$$

a_0 indicating the amplitude of the vibrations, v_0 the maximum value of the velocity of the same vibrations, ϱ the density of the air, and N the number of vibrations pr. sec. In experiment III above, a_0 assumes values as high as $^1/_4$ mm. with a wave-length of ab. 3.4 mm. Corresponding to the said wave-length $N = 10^5$. From $v_0 = 2\pi N a_0$ we get $v_0 = 1.57 \cdot 10^4$ cm./sec. $\backsim$ 157 m./sec. The maximum value δ of the alterations of the density ϱ is given by $\delta = v_0/c$, c denoting the velocity of sound i. e. 340 m./sec. We get $\delta = \frac{157}{340} = 0.46$. Finally the greatest relative alteration of the pressure is $\Delta = \varkappa\delta\ (\varkappa = 1.41)$ thus in the case considered 0.65 or 65 %. The figures will presumably convey an idea as to the comparatively very high intensities occurring in the generator considered.

We may furthermore try to calculate the efficiency of the generator. In doing so we may imagine the stationary vibrations of the oscillator coming into existence by a wave of intensity $\frac{I}{2}$ running down to the bottom of the oscillator, being reflected from the same and afterwards radiated from the oscillator. The sound-energy emitted in this manner would be per sec.

$$2^\circ \qquad W = c \cdot \frac{I}{2} \cdot S,$$

S indicating the area of the cross-section of the oscillator or rather of the vibrating air-column. The intensity I may

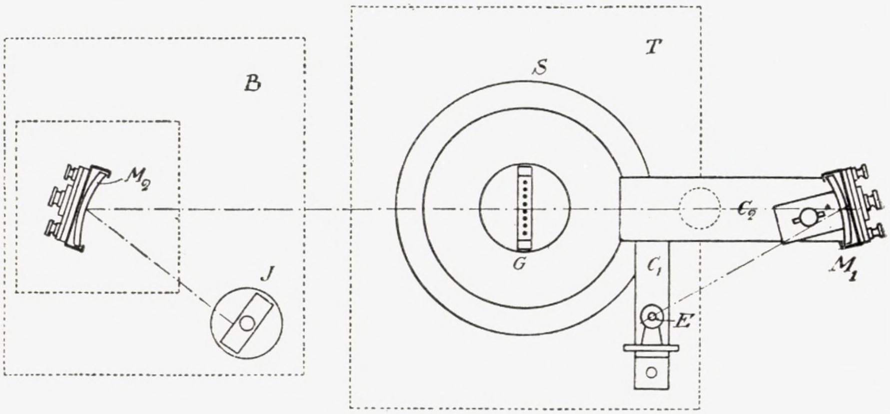

Fig. 20.

be calculated from the expression 1° above. The density ϱ in 1° will approximately be equal to the density of the air in the container from which the air streams, because the air-jet is stopped by the oscillator thus getting its kinetic energy transformed into static energy. In the case considered the over-pressure in the container was 2.74 atm. From this ϱ is found to be 0.00475, giving the value $2.93 \cdot 10^5$ erg/cm^3 for I. We judge the area S through which the energy is radiated to be 1 mm^2. The radiation accordingly assumes the value $34000 \cdot \frac{1}{100} \cdot \frac{2.93}{2} \cdot 10^5 = 0.5 \cdot 10^8$ erg/sec or 0.51 kg m/sec. The work which must be done in compressing 1 kg of air to the over-pressure of 2.74 atm. is

$9.65 \cdot 10^3$ kg. m. provided the air has a temperature of 20° Celcius before the compression. Through the jet-hole of 0.7 mm. diameter, a quantity of air equal to 0.33 g/sec or $0.33 \cdot 10^{-3}$ kg/sec flows at the over-pressure indicated. The compression work thus becomes 3.2 kg m/sec and the efficiency $\frac{0.5}{3.2} \cdot 100 = 16\,\%$, a relatively high value.

Spectrum of the sound-waves.

The experiments indicated above were carried out with the aim of throwing light on the state inside the oscillator. We now proceed to describe some investigations on the sound-waves proper. In order to study these waves a

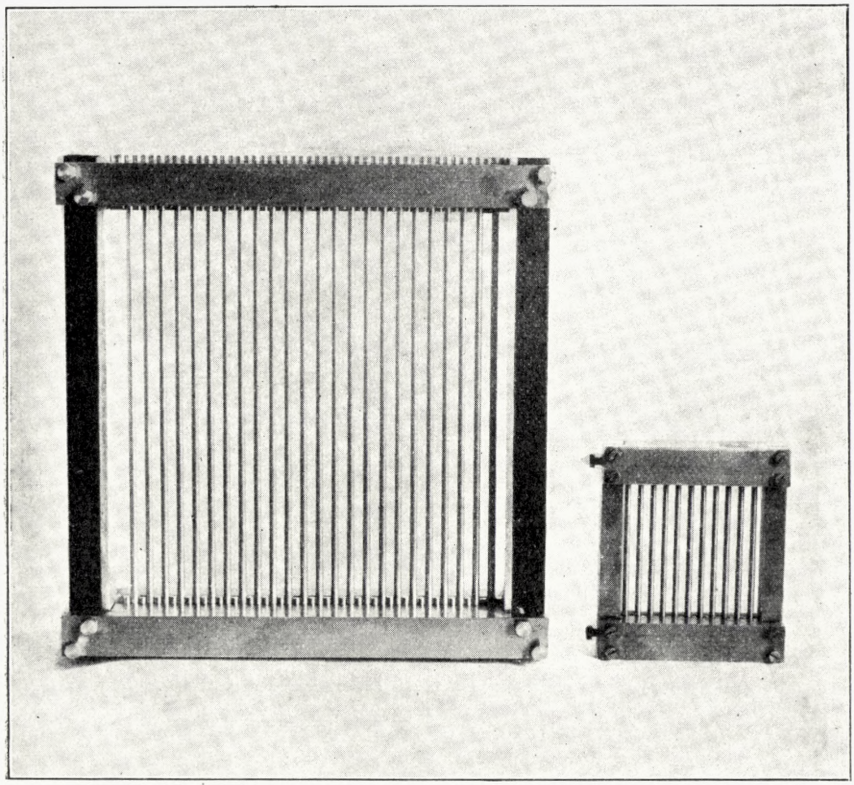

Fig. 21.

grating spectrometer was set up. The spectrometer is shown diagrammatically in fig. 20. *G* indicates the grating placed on the table of a rather large optical spectrometer *S*, from which the telescope and the collimator were removed. To the telescope arm an angular steel-piece formed by the two strips C_1 and C_2 was fastened. To C_2 an adjustable concave mirror M_1 was fixed. The strip C_1 supported the generator *E* adjusted with the mouth of its oscillator in the focus of M_1. The bundle of parallel sound-rays reflected from M_1 was directed against the grating of which fig. 21 shows a photograph (the smaller of the two). When a spectrum was to be studied the telescope arm with the mirror and the generator was turned until the ray bundle deflected by the grating hit the concave-mirror M_2 by which the rays were collected on the slit of the indicator *I*. The latter is shown diagrammatically in fig. 22. To a stem *S* of aluminum-wire a frame *R*, made likewise of aluminum-wire, was attached. To *R* a square piece of paper *F* was fixed, the paper being counterbalanced by a small weight *L*. The stem was suspended by a strip of phosphorbronze *A*. It was fitted with a mirror *M* for the reading of the deflection and furthermore with a damping disk *B* submerged in an oil-bath *O*. The whole system was inclosed in a box, fig. 23, in the wall of which was a slit in front of *F* and a window before *M*. The bundle of sound-rays was concentrated in the plane of the slit. The intensity of the sound was estimated from the pressure on *F* which again was estimated from the deflection of the system. It soon appeared that the production of the spectrum was a very

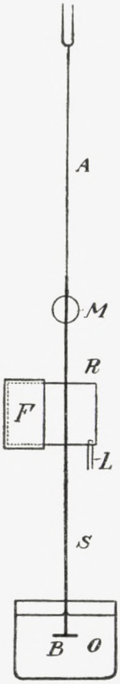

Fig. 22.

difficult task. The difficulties originated mainly from diffraction and reflections inside the box inclosing the indicator. They have not yet been fully conquered. When, never-

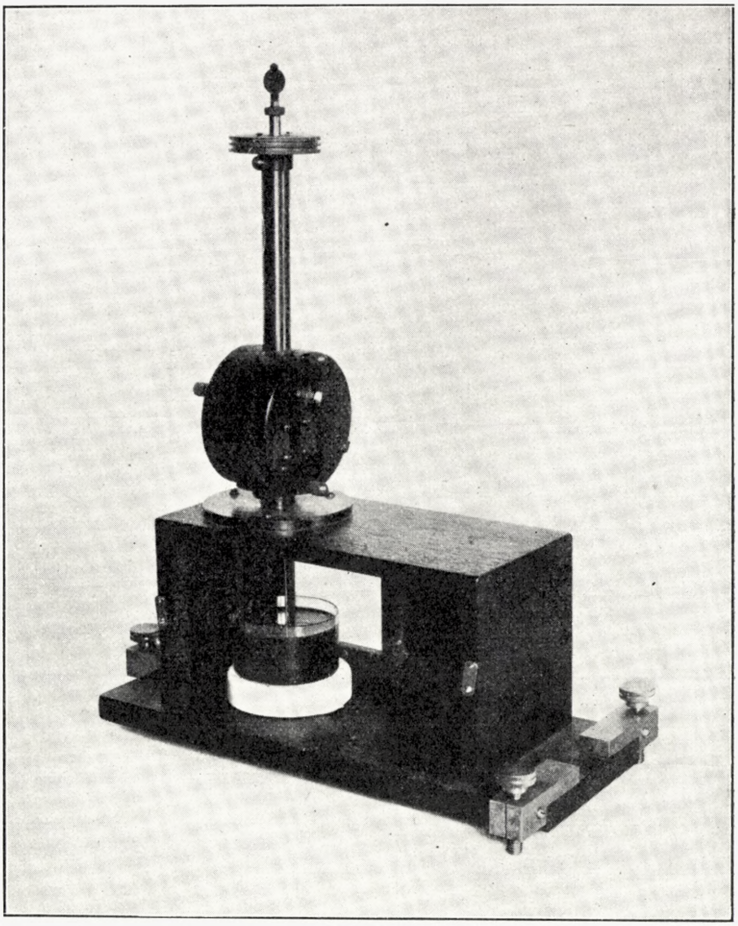

Fig. 23.

theless, results from the spectroscopic investigations will be considered in this paper it is because certain observations, in spite of all imperfection, throw some light on the structure of the waves.

It appeared that the spectra were as a rule rather complex, due partly to the anomalies indicated above. Fig. 24

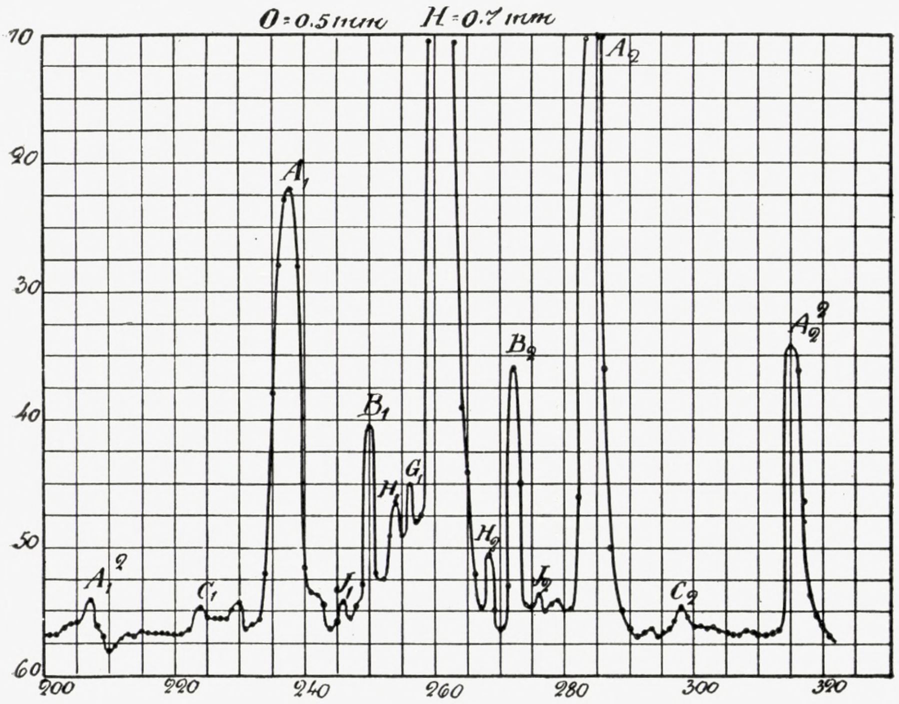
O = 0.5 mm H = 0.7 mm
A_1
B_1
H_1
G_1
J_1
C_1
A_1^2
A_2
B_2
H_2
J_2
C_2
A_2^2
10
20
30
40
50
60
200
220
240
260
280
300
320

Fig. 24.

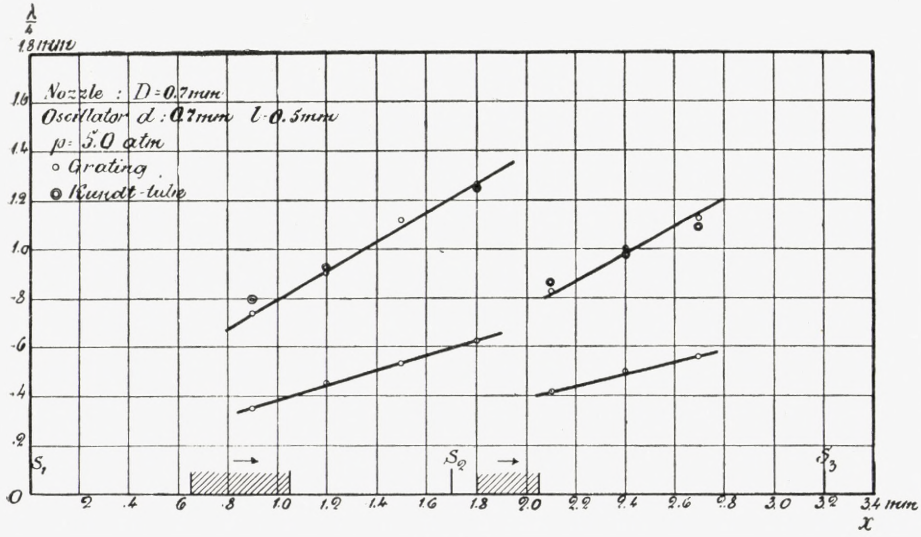
λ/4
1.8 mm
Nozzle : D = 0.7 mm
Oscillator d : 0.7 mm l = 0.5 mm
p = 5.0 atm
○ Grating
◎ Kundt-tube
S_1
S_2
S_3
x
3.4 mm

Fig. 25.

represents a typical spectrum. The oscillator had an aperture of 0.7 mm. and a depth of 0.5 mm. The diameter of the jet hole was 0.7 mm., the over-pressure 3.5 atm. The grating constant was 8.00 mm. In the spectrum, fig. 24, A_1 and A_2 indicate the primary spectra of the fundamental note. The spectrometer gave for the said note the wave-length 3.14 mm. while the Kundt-tube gave 3.10 mm. A_1^2 and A_2^2 represent the secondary spectra of the fundamental note. From B_1 and B_2 the presence of a fairly vigorous note with a wave-length practically equal to half of the fundamental note, namely 1.53 mm., may be concluded. H_1 and H_2, giving $\lambda = 0.97$ mm., seem to represent a note with a frequency thrice that of the fundamental note. In fig. 25 the results of an investigation on the relation between the position of the oscillator and the "spectral-lines" A and B have been represented. It appears that quite the same rectilinear kind of variation as with the Kundt-tube experiment is found. The wave-lengths were also actually measured by means of a Kund-tube. The observations have been entered in the graph. They very nearly coincide with the spectrographic determinations of the fundamental note. The vibrations of half the wave-length, the B-spectra, were of course not revealed by the Kundt-tube. In the graph the boundaries of the jet sections, calculated from Prandtl's expression, and the loop intervals, have furthermore been entered. The complete representation may be compared to fig. 17. Obviously the two pictures convey much the same impression.

Instantaneous photographs of the sound-waves.

From the numerical estimates above it appears that very pronounced pressure gradients must occur in the wave near to the generator. Yet before we were able to make these estimates we realized that the intensity of the waves must be comparatively very high, thus offering a possibility of making the waves visible by means of instantaneous photographs. An attempt was made and the results proved

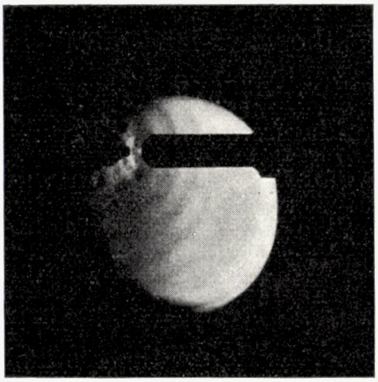

Fig. 26 *a*.

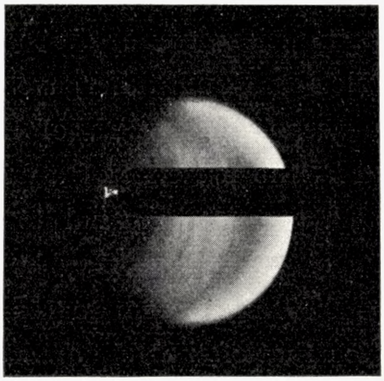

Fig. 26 *b*.

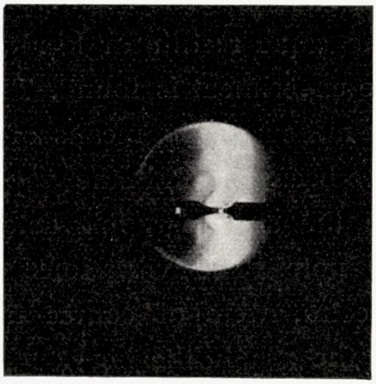

Fig. 26 *c*.

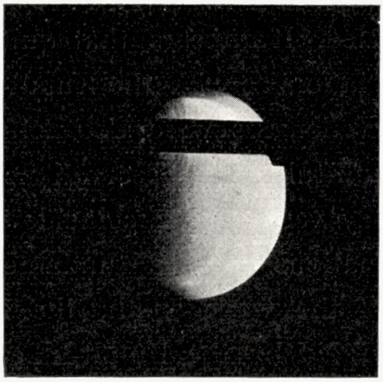

Fig. 26 *d*.

highly illustrative with regard to the structure of the waves. In fig. 26 *a*—*d* four of the photographs have been reproduced, and in the following table results from measurements on the reproduced photographs and others have been entered. It is seen from fig. 26 that the waves mainly appear as rather sharply drawn circles. Especially pronounced is the sharpness in fig. 26 *b*. The distance between

Table V.

No.	Gas	Oscillator d	Oscillator l	λ Phot.	λ Kundt-tube	$N = \frac{340000}{\lambda}$ Phot.	Fig.
		mm	mm	mm	mm		
22	Atm. air.	1	1	3.62	3.3	94000	26 *c*
34	—	1	2	5.20	4.8	65400	
36	—	1	1	3.27	3.2	104000	
37	Hydrogen	1	3	3.92	3.84	86700	26 *a*
38	—	1	3	3.86	3.84	88100	
41	—	0.7	0.5	1.47	—	231000	26 *d*
30	Atm. air.	0.7	3	6.54	7.14	52000	26 *b*

consecutive circles is a complete wave-length, as will appear from table V in which λ in the column marked "Phot" denotes the distance between two consecutive circles, while λ in the column marked "Kundt-tube" indicates the wave-length measured by means of the Kundt-tube.

The structure of the waves.

The instantaneous photographs of fig. 26 can hardly be explained in any other way than by ascribing to the radiated waves a very dissymmetric form, say the form indicated in fig. 27. At any rate waves like those in fig. 27, would give pictures of just the observed character. The serrated shape of the waves corresponds to a sudden expansion of the air in the oscillator. Various observations make it probable that the motion of the air in the oscillator is of this character. With a pulsator the phase of discharge was thus, or might at any rate be, essentially shorter than that of the charge. Presumably we are therefore justified in assuming a similar state of things

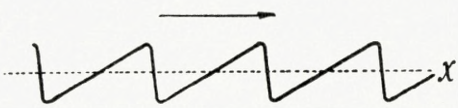

Fig. 27.

with the oscillator. The spectrograms, too, correspond fairly well with the wave-shape of fig. 27. It was a characteristic feature of the spectra that they always contained the octave of the fundamental note. This is just what might be expected from the wave-shape indicated as will be understood from fig. 28. In the said figure the wave A_2

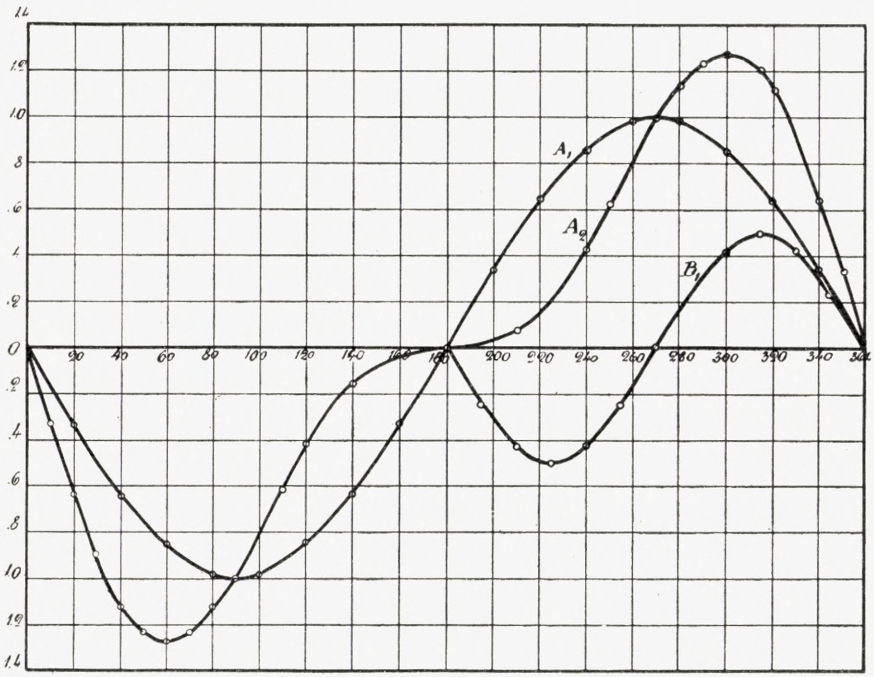

Fig. 28.

has been built up from the two harmonic waves A_1 and B_1, B_1 being the octave of A_1 and having an amplitude which presumably corresponds to the relative intensity of the octave in the grating experiments. It appears that A_2 mainly has the same character as the waves of fig. 27.

According to the general theory of sound-waves a plane wave cannot proceed through the air without changing its form[1]. This

[1] Comp. for instance Lord Rayleigh: Theory of Sound. Sec. Edit. Vol. II, pg. 32.

is due to the fact of the various parts of the wave moving forward with different velocities. In that part of the wave, in which the velocity of the oscillating motion of the particles is v in the direction of the propagation of the wave, the wave proper travels with a velocity $c+v$, c being the average velocity of propagation i. e. "the velocity of sound". On the other hand, the velocity of the wave is $c-v$ at a spot where the particles have the velocity v in the opposite direction of the propagation. The effect of this variation of velocity throughout the wave would just be to change an originally harmonic wave into a wave of the kind shown in fig. 27. It is now a question of some interest whether the deformation considered takes place with the spherical waves radiating from our generator. The velocities of the particles in the waves are certainly of such a size that a distortion would result within a few wave-lengths in the case of a plane wave. The photographs of fig. 26, however, do not at all convey any notion as to a deformation, the circles all having much the same sharpness. We therefore hold the opinion that no such deformation takes place, and might in this connection point out that Lord Rayleigh in his discussion of the deformation suggests that the spherical form of the actual sound waves may account for the obvious discrepancy between the general theory and actual waves, consisting in the latter never being able to reach, far less to pass, the limiting form with a perpendicular front[1].

Most of the experiments described above were carried out as early as in 1920—1921. However, it was not until 1924 that the discussion of the results was brought to an end. Our thanks are due to the Board of The Carlsberg Fund for the means granted us for the accomplishment of the work.

[1] Lord Rayleigh: Theory of sound. Sec. Edit. Vol. II, p. 36.

The Royal Technical Highschool. Physical Laboratory II.
April 1925.

Færdig fra Trykkeriet d. 22. April 1926.